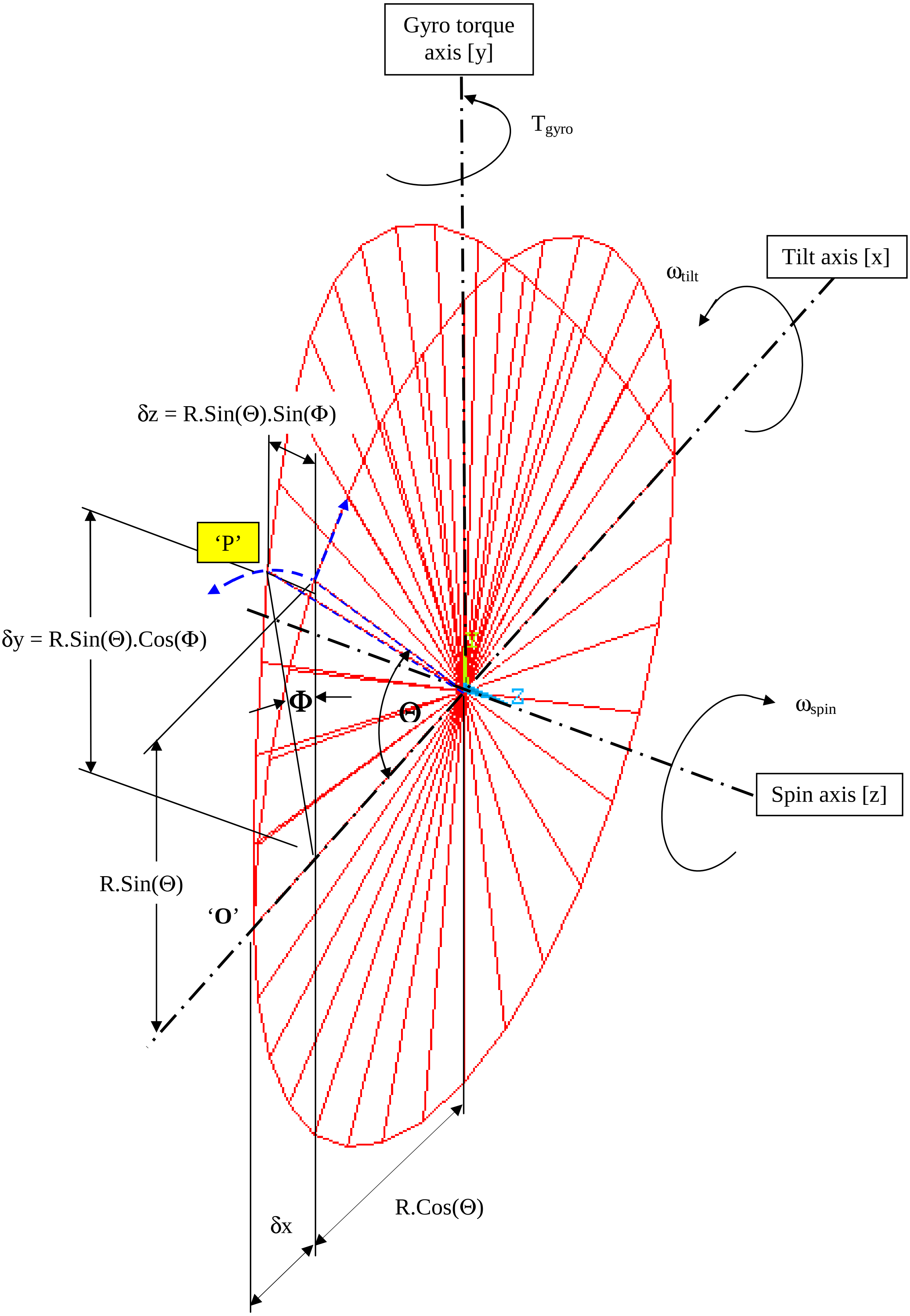
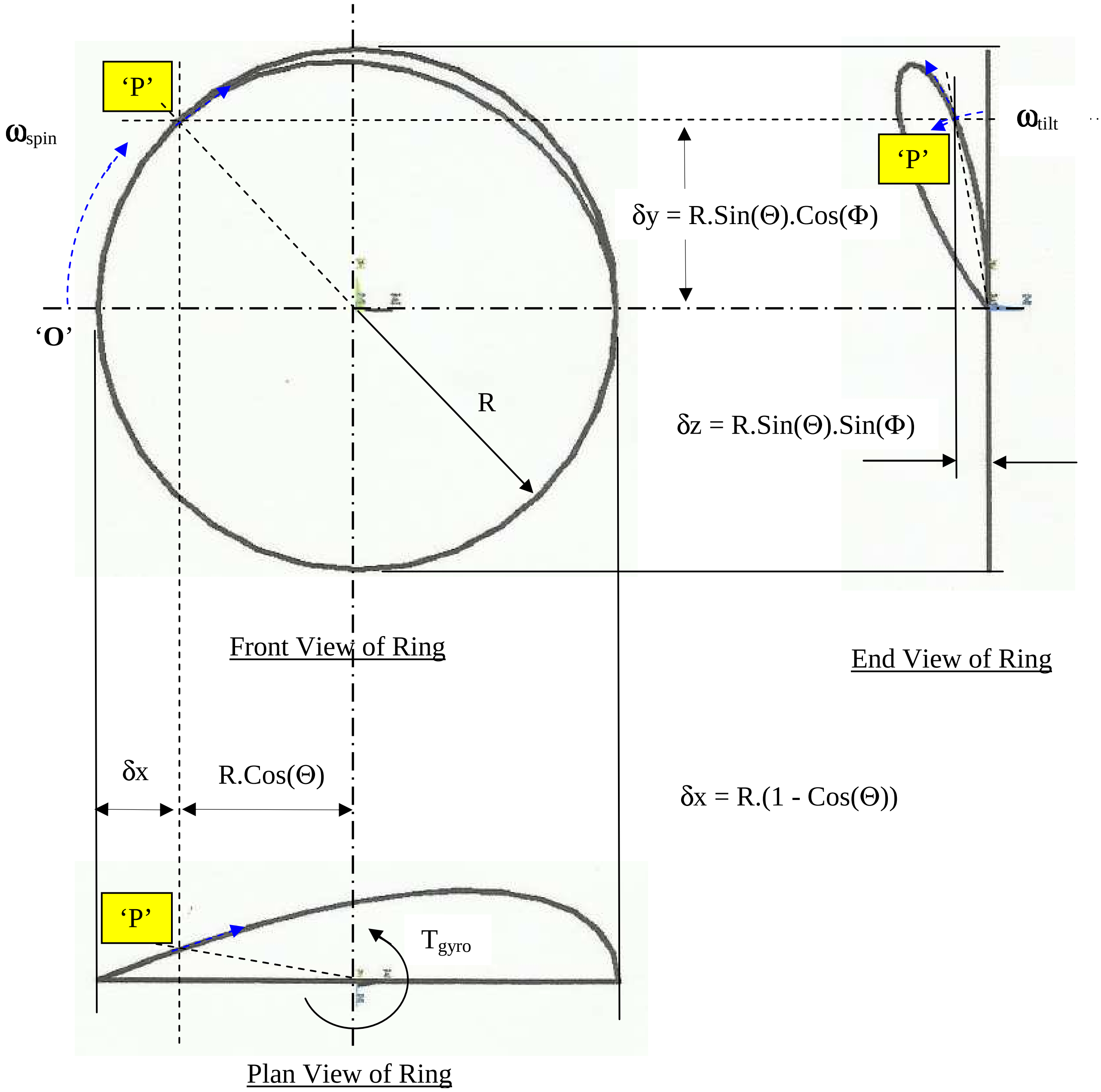
Basic geometry and applied mathematics used in the analysis



The location of point 'P' results from the combined rotation of the ring from point 'O', about the Spin axis by Θ degrees and about the Tilt axis by Φ degrees.



The above three orthogonal views show the locus of a point 'P' on the ring as it simultaneously spins and tilts.

[For clarity, only the first 180^{O} of the locus of point 'P' as it moves from point 'O' are shown in the above three views.]

<u>For pure spin</u> at ω_{spin} rads/sec, without tilting of the spinning ring ((ω_{tilt} = 0 and ϕ = 0),

Displacements, $\delta x = R.(1 - Cos(\omega_{spin}.t)) = R.(1 - Cos(\Theta))$, $\delta y = R.Sin(\Theta)$, $z = 0$
Speed, $dx/dt = -\omega_{spin}.R.Sin(\omega_{spin}.t)$ and $dx/d\Theta = -\omega_{spin}.R.Sin(\Theta)$
Acceleration, $d^2x/dt^2 = -\omega_{spin}^2.R.Cos(\omega_{spin}.t)$ and $d^2x/d\Theta^2 = -\omega_{spin}^2.R.Cos(\Theta)$

For spin at ω_{spin} rads/sec, <u>with simultaneous tilting</u> ($\omega_{tilt} = C.\omega_{spin}$ and $\phi = C.\Theta$),

In plane of spin, displacements, $\delta x = R.(1 - Cos(\omega_{spin}.t)) = R.(1 - Cos(\Theta))$, $\delta y = R.Sin(\Theta).Cos(\phi)$,
[Program line 110]
and <u>Normal</u> to plane of spin, displacement, $\delta z = R.Sin(\Theta).Sin(\phi) = R.Sin(\Theta).Sin(C.\Theta)$

Speed, $dz/d\Theta = \omega_{spin}.R.[Cos(\Theta).Sin(C.\Theta) + C.Sin(\Theta).Cos(C.\Theta)]$
$= \omega_{spin}.R.[Cos(\Theta).Sin(\phi) + C.Sin(\Theta).Cos(\phi)]$ [Program lines 110 & 120]
Acceleration, $d^2z/d\Theta^2 = \omega_{spin}^2.R.[-Sin(\Theta).Sin(\phi) + 2.C.Cos(\Theta).Cos(\phi) - C^2.Sin(\Theta).Sin(\phi)]$
{by product rule, $dz/d\Theta = u.dv/d\Theta + v.du/d\Theta$}
[Program line 130]
<u>Parallel</u> to plane of spin, displacement, $\delta x = R.(1 - Cos(\Theta))$
[Program line 230]
Speed, $dx/d\Theta = -\omega_{spin}.R.Sin(\Theta)$
Acceleration, $d^2x/d\Theta^2 = -\omega_{spin}^2.R.Cos(\Theta)$
[Program lines 240 & 250]

The BASIC numerical integration program, 'GyroTorque.txt', considers a spinning ring of mean radius, R, and mass, M, to be made up of a discrete number of elements, I, each element having a length of $2.\pi.R/I$ and a mass, $\delta M = M/I$.

The above accelerations are calculated at each end of every element, and then averaged to give the accelerations at the centroid of each element, both normal to and parallel to the plane of spin.

By Newton's Second Law, the forces associated with these two accelerations at the centroid of each element are calculated, given by $\delta F = (\delta M \times \text{Acceleration})$.
[Program lines 140 & 260]
By calculating x and z offset distances, $R.Cos(\Theta)$ and $R.Sin(\Theta).Sin(\phi)$ respectively, of every element from the centre of the ring as the ring spins and tilts, the resulting moment ($= \delta F \times$ Offset) about the third (y) axis can then be calculated for every element of the whole ring.
[Program lines 150 & 270]
All of these elemental moments about the third (y) can then be summed for the whole ring to give a calculated value for the gyroscopic torque, T_{gyro}, about the gyroscopic precession axis.
[Program lines 160 & 280]

The result of this numerical integration calculation of the gyroscopic torque can then be directly compared with the formula-derived value of $C.M.\omega_{spin}^2.R^2$.

It is readily evident that the two results are in extremely close agreement, especially when a large number of integration elements are employed and when a small value of C is applied, the ratio of tilt speed to spin speed. The fact that the two results are very slightly different points to the fact that both the formulae-derived value and the numerically integrated value are only approximate.

The two values are approximate in different ways however, as follows:
The approximation inherent in the numerically integrated value is due to the fact that a discrete number of elements are used, whereas the approximation inherent in the formula-derived value is due to the fact that the non-linearity associated with higher tilt speeds is ignored.

Furthermore, the foregoing methodology can be condensed and expressed, after multiplication and substitution, to give the following expression for the total gyroscopic moment:

$$T_{gyro} = M.\omega_{spin}.\omega_{tilt}.R^2.\int_0^{2\pi} Cos(\Theta).\left[2.Cos(\Theta).Cos(C.\Theta) - C.Sin(\Theta).Sin(C.\Theta)\right]d\Theta$$

Using the above expression for T_{gyro}, a separate simplified numerical integration program, 'Integral.txt', has been constructed in which the above definite integral between 2π and zero, representing the full 360^O rotation of the ring, is evaluated.

The numerical result of this integration term is very close to 1.000, thus nominally corroborating the familiar formula, $T_{gyro} = M.\omega_{spin}.\omega_{tilt}.R^2$, for the resulting value of gyroscopic torque.

It is probable that this expression tends to integrate to an exact value of **1** as the number of integration elements tends towards ∞ and as the $(\omega_{tilt}/\omega_{spin})$ speed ratio, C, tends towards zero, although this assertion has not yet been rigorously corroborated by pure mathematics.

In reality though, some degree of tilt speed must always exist in order that a gyroscopic torque arises and therefore, the real value of this coefficient will never be exactly equal to **1**, but will always be slightly less than **1**.
Consequently, it follows that the old familiar text book formula for gyroscopic torque, $T_{gyro} = M.\omega_{spin}.\omega_{tilt}.R^2$, can never be perfectly accurate.

It is recommended that the assertions implicit in this paper be practically corroborated by experimental measurements of the value of gyroscopic torque developed for various values of C, the angular speed ratio, $(\omega_{tilt}/\omega_{spin})$.